Large *j* behaviour of dipole cosmology transition amplitudes

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# Large *j* behaviour of dipole cosmology transition amplitudes

### Jacek Puchta

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### Tux, 27th of Februart 2013 EFI winter conference on canonical and covariant loop quantum gravity

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### Introduction Motivation Saddle Point Approximation The lorentzian polyhedra propagator - preliminary analysis Definition and basic properties Symmetries of the integrand Strategy of integration Integrand's smoothness check $f_m^{(j)}(\eta)$ $\frac{1}{J}\ln\left(f_m^{(j)}(\eta)\right)$ $\phi_{\vec{m}}(\eta, J)$ Hessian Results and summary Result Summary









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#### Introduction

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### The aim

to investigate the properties of Lorentzian Polyhedra Propagator:

$$\mathbb{T} = \int_{SL(2,\mathbb{C})} \mathrm{d}g Y^{\dagger} g Y$$

in large j limit

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in large *j* limit

### **Motivation**

• DC amplitude is proportional to  $\int dg \prod_{i=1}^{4} \langle \vec{n}_i | Y^{\dagger} g Y | \vec{n}_i \rangle_i$ 

[Bianchi, Rovelli, Vidotto: Phys.Rev.D82 (2010)], [Vidotto: Class.Quantum Grav.28 (2011)], [Borja, Garay, Vidotto: SIGMA 8 (2012)]



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► 2-vertex-and-1-edge DC:  $\int dg dg' \prod_{i=1}^{4} \langle \vec{n}_i | Y^{\dagger}gY Y^{\dagger}g'Y | \vec{n}_i \rangle_j \text{ [JP, in progress]}$ 



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- ► 2-vertex-and-1-edge DC:  $\int dg dg' \prod_{i=1}^{4} \langle \vec{n}_i | Y^{\dagger}gY Y^{\dagger}g'Y | \vec{n}_i \rangle_j \text{ [JP, in progress]}$
- ► radiative corrections of "melonic" graph proportional to  $\log \Lambda \cdot \mathbb{T}^2$  [Riello: arXiv:1302.1781 (2013)]





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# The SPA method

We estimate the following integral for  $\Lambda \gg 1$  by the value of integrand in the critical point  $x_0$  of f(x)

$$\int \mathrm{d}x \ g(x) e^{-\Lambda f(x)} = \sqrt{\frac{(2\pi)}{\Lambda |f''(x_0)|}} g(x_0) e^{-\Lambda f(x_0)}$$

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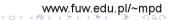
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for multidimensional integrals:

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$$\int \mathrm{d}^{n} x \ g(x) e^{-\Lambda f(x)} = \sqrt{\left(\frac{2\pi}{\Lambda}\right)^{n}} \left(\left|\frac{\partial^{2} f}{\partial x^{2}}\right|_{x_{0}}\right)^{-1/2} g(x_{0}) e^{-\Lambda f(x_{0})}$$
(2)

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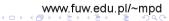
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(2)

### Note!

 $\nabla f(x)$  must vanish in  $x_0$ , i.e.  $x_0$  must not be the extremum, where  $\nabla f(x)$  is discontinues.







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# Not obvious decomposition of integrand

What if the function we integrate does not have an obvious decomposition into  $g(x)e^{-\Lambda f(x)}$ ?

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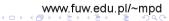
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What if the function we integrate does not have an obvious decomposition into  $g(x)e^{-\Lambda f(x)}$ ?

Consider  $F(\Lambda) = \int d^x \Phi(x, \Lambda)$ . Let's assume, that the integrand  $\Phi(x, \Lambda)$  has appropriate asymptotic behaviour, but we don't know its decomposition into f(x) and g(x). In such a case we need to investigate the function

$$\phi(x) := \lim_{\Lambda \to \infty} \frac{1}{\Lambda} \ln(\Phi(x, \Lambda))$$
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we will call it *the exponent part* of the integrand.

The SPA formula will be true for critical points and Hessian matrix of  $\phi(x)$ .







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$$f_m^{(j)}(\eta) \\ \frac{1}{J} \ln \left( f_m^{(j)}(\eta) \\ \phi_{\vec{m}}(\eta, J) \right) \\ \text{Hessian}$$

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# Definition

### Def: Lorentzian polyhedra propagator

Given a set of spins  $j_1, \ldots, j_N$  we define an operator

$$\mathbb{T} := \int_{SL(2,\mathbb{C})} \mathrm{d}g \; \left[ Y^{\dagger} g Y \right]^{(j_1 \otimes \cdots \otimes j_N)}$$

acting on  $\mathcal{H}_{i_1} \otimes \cdots \otimes \mathcal{H}_{i_N}$ 



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acting on  $\mathcal{H}_{j_1}\otimes\cdots\otimes\mathcal{H}_{j_N}$ 

We can consider its matrix elements in the  $|m\rangle_i$  basis:

$$\begin{split} \mathbb{T}_{m_{1}\cdots m_{N}}^{m_{1}'\cdots m_{N}'} &:= \int_{SL(2,\mathbb{C})} \mathrm{d}g \,\langle m_{1}, \dots, m_{N} | \, Y^{\dagger}gY \, | m_{1}', \dots, m_{N}' \rangle_{j_{1}\otimes \cdots \otimes j_{N}} \\ &= \int_{SL(2,\mathbb{C})} \mathrm{d}g \prod_{i=1}^{N} \langle m_{i} | \, Y^{\dagger}gY \, | m_{i}' \rangle_{j_{i}} \\ &= \int_{SL(2,\mathbb{C})} \mathrm{d}g \prod_{i=1}^{N} D^{(\gamma j_{i}, j_{i})}(g)_{j_{i}, m_{i}'}^{j_{i}, m_{i}'} \end{split}$$

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It is easy to check, that  $\mathbb T$  acts non-trivially only on the invariant subspace of  $\mathcal H_{j_1}\otimes \cdots \otimes \mathcal H_{j_N}$ :

$$\mathbb{T} = \int_{SL(2,\mathbb{C})} \mathrm{d}g \ Y^{\dagger}gY = \int_{\mathbb{R}^{3} \times SU(2)} \mathrm{d}k \mathrm{d}u \ Y^{\dagger}kuY$$
$$= \int_{\mathbb{R}^{3}} \mathrm{d}k \ Y^{\dagger}kY \int_{SU(2)} u\mathrm{d}u = \hat{A} \cdot P_{\mathrm{Inv}}$$

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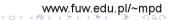
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Thus

$$\mathbb{T} = P_{\mathrm{Inv}} \cdot \hat{B} \cdot P_{\mathrm{Inv}}$$

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Thus

$$\mathbb{T} = P_{\mathrm{Inv}} \cdot \hat{B} \cdot P_{\mathrm{Inv}}$$

So it's enough to study the matrix elements between the SU(2) invariants:

$$\mathbb{T}_{\iota\iota'} = \int_{\mathit{SL}(2,\mathbb{C})} \mathrm{d}g \, \langle \iota | \, Y^{\dagger}gY \, | \iota' 
angle$$







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## Symmetries of the integrand 1 - SU(2)

We need to integrate the function  $\Phi_{\iota\iota'}(g, J) := \langle \iota | Y^{\dagger}gY | \iota' \rangle$  on  $SL(2, \mathbb{C})$ , where  $J = \max_{i=1,...,N}(j_i)$ . We anticipate that the critical point will be in  $g = \mathbf{1}$ . Let us study the behaviour of  $\Phi$  close to  $g = \mathbf{1}$ .

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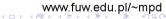
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There are six-dimensional basis vector fields on  $SL(2, \mathbb{C})$  given by the generators of rotations  $J_i$  and generators of boosts  $K_i$  (i = 1, 2, 3).

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There are six-dimensional basis vector fields on  $SL(2, \mathbb{C})$  given by the generators of rotations  $J_i$  and generators of boosts  $K_i$  (i = 1, 2, 3).

It's straightforward to see, that  $J_i \Phi_{\iota\iota'}(g) \equiv 0$ Indeed:  $J_i$  are SU(2) generators, thus they commute with the Y map, and  $J_i |\iota\rangle = 0$ , so

$$\langle \iota | \ Y^{\dagger}g J_{i} Y | \iota' 
angle = \langle \iota | \ Y^{\dagger}g Y J_{i} | \iota' 
angle = 0$$







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Let's now consider a z-boost  $Y^{\dagger}e^{\eta K_3}Y$ . Since  $[K_3, J_3] = 0$ , it's convenient to consider it in the  $|m\rangle_i$  basis.

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Let's now consider a z-boost  $Y^{\dagger}e^{\eta K_3}Y$ . Since  $[K_3, J_3] = 0$ , it's convenient to consider it in the  $|m\rangle_i$  basis.

Let's define the function  $f_m^{(j)}(\eta)$ 

$$\langle m | Y^{\dagger} e^{\eta K_3} Y | m' 
angle_j =: \delta_{m,m'} f_m^{(j)}(\eta)$$

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We decompose the invariant tensors in  $|m\rangle_i$  basis

$$|\iota\rangle = \sum_{\{m_i\}} \iota_{m_1\cdots m_N} |m_1,\ldots,m_N\rangle_{j_1\otimes\cdots\otimes j_N}$$

and thus

$$\Phi_{\iota,\iota'}(e^{\eta K_3}) = \sum_{\{m_i\}} \iota^{m_1\cdots m_N} \iota'_{m_1\cdots m_N} \Phi^{m_1\cdots m_N}_{m_1\cdots m_N}(e^{\eta K_3})$$

where  $\Phi_{m_1 \cdots m_N}^{m_1 \cdots m_N}(e^{\eta K_3}) = \prod_{i=1}^N f_{m_i}^{(j_i)}(\eta)$ 



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where  $\Phi_{m_1 \cdots m_N}^{m_1 \cdots m_N}(e^{\eta K_3}) = \prod_{i=1}^N f_{m_i}^{(j_i)}(\eta)$ 

### Note that

since  $J_i |\iota\rangle = 0$ , only the terms with  $\sum_{i=0}^{N} m_i = 0$  counts.

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Consider now a boost in arbitrary direction  $\vec{n}$ .

Since

$$e^{\eta \vec{n} \cdot \vec{K}} = e^{u^{-1}\eta K_3 u} = u^{-1} e^{\eta K_3} u$$

for some  $u \in SU(2)$ ,

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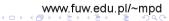
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$$\Phi_{\iota\iota'}(e^{\eta\vec{n}\cdot\vec{K}}) = \langle \iota | Y^{\dagger}u^{-1}e^{\eta K_3}uY | \iota' \rangle$$

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for some  $u \in SU(2)$ , the value of  $\Phi_{\iota\iota'}(e^{\eta \vec{n}\cdot \vec{K}})$  is given by

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Consider now a boost in arbitrary direction  $\vec{n}$ .

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$$\begin{split} \Phi_{\iota\iota'}(e^{\eta\vec{n}\cdot\vec{K}}) &= \langle \iota | Y^{\dagger}u^{-1}e^{\eta K_3}uY | \iota' \rangle = \langle \iota | u^{-1}Y^{\dagger}e^{\eta K_3}Yu | \iota' \rangle \\ &= \langle \iota | Y^{\dagger}e^{\eta K_3}Y | \iota' \rangle = \Phi_{\iota\iota'}(e^{\eta K_3}) \end{split}$$

Thus the behaviour of  $\Phi_{\iota\iota'}(e^{\eta K_3})$  and  $f_m^{(j)}(\eta)$  is crucial in further calculation.







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# Strategy of integration 1: parametrisation of $SL(2,\mathbb{C})$

We need to integrate  $\int_{SL(2,\mathbb{C})} \mathrm{d}g \, \Phi_{\iota\iota'}(e^{\eta(g)K_3}).$ 

Since the integrand depend only on  $\eta$ , one may be tempted to use the decomposition  $g = u_1^{-1} e^{\eta K_3} u_2$  and the measure

$$\int_{SU(2)\times SU(2)\times \mathbb{R}_+} \mathrm{d} u_1 \mathrm{d} u_2 \frac{\sinh^2 \eta}{4\pi} \mathrm{d} \eta \Phi_{\iota\iota'}(e^{\eta K_3})$$

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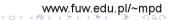
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however we anticipate that the maximum of  $\Phi$  is  $\eta=$  0, where the parametrisation breaks down.

Thus let's parametrise  $SL(2,\mathbb{C})$  with  $(u, \vec{x}) \in SU(2) \times \mathbb{R}^3$ :

$$g(u, \vec{x}) := u n_{\vec{x}}^{-1} e^{|\vec{x}|K_3} n_{\vec{x}}$$

where  $n_{\vec{x}} \in SU(2)$  is such a rotation, that  $n_{\vec{x}}^{-1} |\vec{x}| L_3 n_{\vec{x}} = \vec{x} \cdot \vec{L}$ , i.e.

$$n_{\vec{x}} = \begin{pmatrix} \cos \frac{\theta(\vec{x})}{2} & -e^{i\phi(\vec{x})} \sin \frac{\theta(\vec{x})}{2} \\ e^{-i\phi(\vec{x})} \sin \frac{\theta(\vec{x})}{2} & \cos \frac{\theta(\vec{x})}{2} \end{pmatrix}$$

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$$\mathrm{d}g = \mathrm{d}u_1 \,\mathrm{d}u_2 \,\frac{\sinh^2\eta}{4\pi} \mathrm{d}\eta = \mathrm{d}u \,\mathrm{d}^3 x \,\mu(\vec{x})$$

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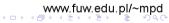
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$$dg = du_1 du_2 \frac{\sinh^2 \eta}{4\pi} d\eta = du d^3 x \mu(\vec{x})$$
$$= du \frac{1}{(4\pi)^2} d\phi(\vec{x}) \sin \theta(\vec{x}) d\theta(\vec{x}) \sinh^2 \eta(\vec{x}) d\eta(\vec{x})$$

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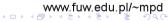
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Putting  $\mathrm{d}^3x=|\vec{x}|^2\sin\theta\mathrm{d}\theta\mathrm{d}\phi$  and  $\eta=|\vec{x}|$  we get

$$\mu(\vec{x}) = \left(\frac{\sinh\eta}{4\pi\eta}\right)^2$$

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$$\mu(\vec{x}) = \left(\frac{\sinh\eta}{4\pi\eta}\right)^2$$

## Note that

There are two properties of the measure important for the SPA method:

• 
$$\lim_{\vec{x}\to 0} \mu(\vec{x}) = \frac{1}{(4\pi)^2}$$

▶  $\lim_{J\to\infty} \frac{1}{J} \ln [\mu(\vec{x})] = 0$  - thus it does not effect the behaviour of the exponent part of the integrand.

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# Strategy of integration 3: SPA and Hessian

We expect the critical point to be  $\vec{x}_0 = 0$ . If the exponent part  $\phi_{\mu\nu'}(\vec{x})$  of the integrand  $\Phi_{\mu\nu'}(\vec{x})$  is smooth (and so is its derivative) in  $\vec{x}_0$ , the integral equals

$$\mathbb{T}_{\iota\iota'} = \int_{SL(2,\mathbb{C})} \mathrm{d}g \, \Phi_{\iota\iota'}(g) = \left(\frac{2\pi}{J}\right)^{\frac{3}{2}} \frac{1}{\sqrt{|H|}} \mu(0) \Phi_{\iota\iota'}(0) \left(1 + O\left(\frac{1}{J}\right)\right)$$

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where H is the Hessian matrix of  $\phi_{\mu\nu'}(\vec{x}) := \lim_{J\to\infty} \frac{1}{I} \ln \Phi_{\mu\nu'}(\vec{x})$ 

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# Strategy of integration 3: SPA and Hessian

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where *H* is the Hessian matrix of  $\phi_{\iota\iota'}(\vec{x}) := \lim_{J\to\infty} \frac{1}{J} \ln \Phi_{\iota\iota'}(\vec{x})$ 

### Hessian of $\phi$

Since  $\phi_{\iota\iota'}$  is spherically symmetric (so it is a function of one variable  $\eta$ ), we can express it's Hessian for  $\eta \to 0$  as

$$\det \left[H_{ij}\right]_{\eta=0} = \det \left[\frac{1}{2} \left(\partial_i \partial_j \eta^2\right) \frac{\mathrm{d}^2 \phi}{\mathrm{d}\eta^2}\right] = \det \left[\delta_{ij} \frac{\mathrm{d}^2 \phi}{\mathrm{d}\eta^2}\right] = \left(\frac{\mathrm{d}^2 \phi}{\mathrm{d}\eta^2}\right)^3$$







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# The Lorentzian polyhedra propagator

Recall now, that

$$\Phi_{\iota\iota'}(0) = \langle \iota | Y^{\dagger} e^{0 \cdot K_3} Y | \iota' \rangle = \langle \iota | \iota' \rangle = \delta_{\iota\iota'}$$

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# The Lorentzian polyhedra propagator

So neglecting the  $\frac{1}{J}$  terms the operator  $\mathbb{T}$  is

$$\mathbb{T}_{\iota\iota'} = \left(\frac{2\pi}{J}\right)^{\frac{3}{2}} \left(\frac{\mathrm{d}^2\phi_{\iota\iota'}}{\mathrm{d}\eta^2}\right)^{-\frac{3}{2}} \frac{1}{(4\pi)^2} \ \delta_{\iota\iota'}$$

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Now we will check smoothness of the exponent part of integrand, and calculate the Hessian.







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# Outline

# Integrand's smoothness check







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### Integrand's smoothness check



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# Strategy

We are going to investigate the function

$$\phi_{\vec{m}}(\eta, J) = \frac{1}{J} \ln \left( \prod_{i=1}^{N} f_{m_i}^{(j_i)}(\eta) \right) = \sum_{i=1}^{N} \frac{1}{J} \ln \left( f_{m_i}^{(j_i)}(\eta) \right)$$

in the limit  $J \gg 1$  and  $\eta \ll 1$ .

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Hessian









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in the limit  $J \gg 1$  and  $\eta \ll 1$ .

We will do it by finding a compact form of  $f_m^{(j)}(\eta)$  and analysing its Taylor series.

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$$\begin{array}{l} f_{m}^{(j)}(\eta) \\ \frac{1}{J} \ln \left( f_{m}^{(j)}(\eta) \right) \\ \phi_{\overrightarrow{m}}(\eta, J) \end{array}$$

Hessian

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$$f_m^{(j)}(\eta) = \langle m | Y^{\dagger} e^{\eta K_3} Y | m \rangle_j = D^{(\gamma j, j)} (e^{\eta K_3})_{j,m}^{j,m}$$

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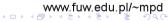
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$$f_m^{(j)}(\eta) = \langle m | Y^{\dagger} e^{\eta K_3} Y | m \rangle_j = D^{(\gamma j, j)} (e^{\eta K_3})_{j, m}^{j, m}$$

Thanks to Y maps, which make the parameters (p, k) of primary series dependent on j, and which pick the lowest spin subspace of (p, k), these matrix elements has rather simple form

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 $\begin{array}{c} f_{m}^{(j)}(\eta) \\ \frac{1}{J} \ln \left( f_{m}^{(j)}(\eta) \right) \\ \phi_{m}^{\rightarrow}(\eta, J) \end{array}$ 

Hessian





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Thanks to Y maps, which make the parameters (p, k) of primary series dependent on j, and which pick the lowest spin subspace of (p, k), these matrix elements has rather simple form

$$\begin{aligned} f_m^{(j)}(\eta) &= (2j+1) \binom{2j}{j+m} e^{-m\eta} e^{i\gamma j\eta} e^{-(j+1)\eta} \\ &\int_0^1 \mathrm{d}x \, x^{j+m} (1-x)^{j-m} \left(1 - \left(1 - e^{-2\eta}\right) x\right)^{i\gamma j - (j+1)} \end{aligned}$$

[Ruhl (1970)]

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ntegrand's smoothness check  $f^{(i)}(\eta)$ 

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[Ruhl (1970)]

(simple when compared to the general  $SL(2, \mathbb{C})$  representation's matrix elements)







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# The $f_m^{(j)}(\eta)$ function in hypergeometric representation

$$\begin{aligned} f_m^{(j)}(\eta) &= (2j+1) \binom{2j}{j+m} e^{-m\eta} e^{i\gamma j\eta} e^{-(j+1)\eta} \\ &\int_0^1 \mathrm{d}x \, x^{j+m} (1-x)^{j-m} \left(1 - (1-e^{-2\eta}) \, x\right)^{i\gamma j - (j+1)} \end{aligned}$$

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Recalling the integral definition of the Hypergeometric Function of 2nd kind

$$_{2}F_{1}(a,b,c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_{0}^{1} \mathrm{d}t \, t^{b-1} (1-t)^{c-b-1} (1-zt)^{-a}$$

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 $\begin{array}{c} f_{m}^{(j)}(\eta) \\ \frac{1}{J} \ln \left( f_{m}^{(j)}(\eta) \right) \\ \phi_{m}^{\rightarrow}(\eta, J) \end{array}$ 

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we get

$$f_m^{(j)}(\eta) = e^{-(j+m+1)\eta} e^{ij\gamma\eta} \\ {}_2F_1\left((j+m+1), (j+1-ij\gamma), (2j+2); (1-e^{-2\eta})\right)$$

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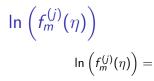
**FNP** 



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 $f^{(j)}(\eta)$ 



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 $\frac{\frac{1}{J}}{\int_{m}} \ln \left( f_{m}^{(j)}(\eta) \right) \phi_{\overrightarrow{m}}(\eta, J)$ 







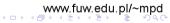


 $\ln\left(f_m^{(j)}(\eta)\right) = -(j+m+1)\eta$ 

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 $\frac{\frac{1}{J}}{\int_{m}} \ln \left( f_{m}^{(j)}(\eta) \right) \phi_{\overrightarrow{m}}(\eta, J)$ 









$$\ln\left(f_m^{(j)}(\eta)\right) = -(j+m+1)\eta + ij\gamma\eta$$

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$$\ln\left(f_m^{(j)}(\eta)\right) = -(j+m+1)\eta + ij\gamma\eta + \psi(\eta)$$

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$$\ln\left(f_m^{(j)}(\eta)
ight)=\,-\,(j+m+1)\eta+ij\gamma\eta+\psi(\eta)$$

$$\psi(\eta) := \ln \left[ {}_{2}F_{1} \left( j + m + 1, \ j + 1 - ij\gamma, \ 2j + 2; \ 1 - e^{-2\eta} \right) 
ight]$$

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 $\frac{\frac{1}{J}}{\int_{m}} \ln \left( f_{m}^{(j)}(\eta) \right) \phi_{\overrightarrow{m}}(\eta, J)$ 









$$\ln \left( f_m^{(j)}(\eta) \right) = -(j+m+1)\eta + ij\gamma\eta + \psi(\eta)$$
$$\psi(\eta) := \ln \left[ {}_2F_1 \left( j+m+1, \ j+1-ij\gamma, \ 2j+2; \ 1-e^{-2\eta} \right) \right]$$

Note that the fourth argument of  $_2F_1$  is small for  $\eta$  close to 0. Indeed,  $1 - e^{-2\eta} = 2\eta + O(\eta^2)$ .



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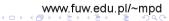
 $\frac{1}{i} \ln \left( f_m^{(j)}(\eta) \right)$ 

Hessian









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Note that the fourth argument of  $_2F_1$  is small for  $\eta$  close to 0. Indeed,  $1 - e^{-2\eta} = 2\eta + O(\eta^2)$ .

Let's now recall the series definition of  $_2F_1$ :

$$_{2}F_{1}(a,b,c;z) := \sum_{k=0}^{\infty} \frac{a^{\overline{k}}b^{\overline{k}}}{c^{\overline{k}}k!} z^{k}$$

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For z close to 0 we Taylor expand it obtaining

$$_{2}F_{1}(a, b, c; z) = 1 + \frac{ab}{c}z + \frac{a(a+1)b(b+1)}{2c(c+1)}z^{2} + O(z^{3}) = 1 + \epsilon$$

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For z close to 0 we Taylor expand it obtaining

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Thus we can apply the Taylor expansion to  $\ln \left[ {}_{2}F_{1}(a, b, c; z) \right]$ 

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# Taylor expansion of $\ln [_2F_1(a, b, c; z)]$

$$\psi(\eta) := \ln \left[ {}_{2}F_{1}\left( j+m+1, \; j+1-ij\gamma, \; 2j+2; \; 1-e^{-2\eta} 
ight) 
ight]$$

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# Taylor expansion of $\ln [_2F_1(a, b, c; z)]$

$$\psi(\eta) := \ln \left[ {}_2F_1 \left( j + m + 1, \; j + 1 - ij\gamma, \; 2j + 2; \; 1 - e^{-2\eta} 
ight) 
ight]$$

$$= \frac{2(j+1)(j+m+1)}{2j+1}\eta - i\frac{2\gamma j(j+m+1)}{2j+1}\eta \\ - \frac{(j-m)(j+m+1)\left[(1-\gamma^2)j+1\right]j}{(j+1)(2j+1)^2}\eta^2 \\ + i\frac{\gamma j(j-m)(j+m+1)}{(j+1)(2j+1)}\eta^2 + O(\eta^3)$$

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Separating into m dependent and m independent part:

$$\begin{split} \psi(\eta) &= \frac{2(j+1)^2}{2j+1}\eta - i\frac{2\gamma j(j+1)}{2j+1}\eta \\ &- \frac{j^2 \left[(1-\gamma^2)j+1\right]}{(2j+1)^2}\eta^2 + i\frac{\gamma j^2}{(2j+1)}\eta^2 + O(\eta^3) \\ &+ \frac{2(j+1)}{2j+1}\eta \cdot m - i\frac{2\gamma j}{2j+1}\eta \cdot m \\ &+ \frac{\left[(1-\gamma^2)j+1\right]j}{(j+1)(2j+1)^2}\eta^2 \cdot m - i\frac{\gamma j}{(j+1)(2j+1)}\eta^2 \cdot m \\ &+ \frac{\left[(1-\gamma^2)j+1\right]j}{(j+1)(2j+1)^2}\eta^2 \cdot m^2 - i\frac{\gamma j}{(j+1)(2j+1)}\eta^2 \cdot m^2 \end{split}$$

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Extracting the leading order terms (in j):

$$\psi(\eta) = j\eta \left[1 + O\left(j^{-1}\right)\right] - i\gamma j\eta \left[1 + O\left(j^{-1}\right)\right]$$

$$- \, rac{j(1-\gamma^2)}{4} \eta^2 \left[ 1 + O\left(j^{-1}
ight) 
ight] \ + \ i rac{\gamma j}{2} \eta^2 \left[ 1 + O\left(j^{-1}
ight) 
ight]$$

$$+ \eta \cdot m \left[1 + O\left(j^{-1}\right)\right] - i\gamma\eta \cdot m \left[1 + O\left(j^{-1}\right)\right]$$

$$+ \frac{1 - \gamma^{-}}{4j} \eta^{2} \cdot m^{2} \left[ 1 + O\left(j^{-1}\right) \right] - i \frac{\gamma}{2j} \eta^{2} \cdot m^{2} \left[ 1 + O\left(j^{-1}\right) \right]$$

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$$\frac{f_{m}(j)}{m}(\eta)$$

$$\frac{1}{j}\ln\left(f_{m}(j)(\eta)\right)$$

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Extracting the leading order terms (in j):

$$\psi(\eta) = j\eta \left[1 + O\left(j^{-1}\right)\right] - i\gamma j\eta \left[1 + O\left(j^{-1}\right)\right]$$

$$- \frac{j(1-\gamma^2)}{4} \eta^2 \left[ 1 + O\left(j^{-1}\right) \right] + i \frac{\gamma j}{2} \eta^2 \left[ 1 + O\left(j^{-1}\right) \right]$$

$$+ \eta \cdot m \left[1 + O(j^{-1})\right] - i\gamma\eta \cdot m \left[1 + O(j^{-1})\right]$$

+ 
$$\frac{1-\gamma^2}{4j}\eta^2 \cdot m \left[1+O(j^{-1})\right] - i\frac{\gamma}{2j}\eta^2 \cdot m \left[1+O(j^{-1})\right]$$

$$+ rac{1-\gamma^2}{4j} \eta^2 \cdot m^2 \left[1+O\left(j^{-1}
ight)
ight] \ - \ irac{\gamma}{2j} \eta^2 \cdot m^2 \left[1+O\left(j^{-1}
ight)
ight]$$

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the terms proportional to  $m\eta^2$  are negligible with respect to terms  $m^2\eta^2.$ 







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$$\frac{f_{m}^{(j)}(\eta)}{\int_{J} \ln \left(f_{m}^{(j)}(\eta)\right)}$$

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Extracting the leading order terms (in j):

$$\psi(\eta) = j\eta \left[1 + O\left(j^{-1}\right)\right] - i\gamma j\eta \left[1 + O\left(j^{-1}\right)\right]$$

$$-\frac{j(1-\gamma^{2})}{4}\eta^{2}\left[1+O(j^{-1})\right] + i\frac{\gamma j}{2}\eta^{2}\left[1+O(j^{-1})\right]$$
$$+\eta \cdot m\left[1+O(j^{-1})\right] - i\gamma\eta \cdot m\left[1+O(j^{-1})\right]$$

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$$+ rac{1-\gamma^2}{4j}\eta^2 \cdot m^2 \left[1+O\left(j^{-1}
ight)
ight] - irac{\gamma}{2j}\eta^2 \cdot m^2 \left[1+O\left(j^{-1}
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ight]$$

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the terms proportional to  $m\eta^2$  are negligible with respect to terms  $m^2\eta^2.$ 





Hessian

Back to  $\ln\left(f_m^{(j)}(\eta)\right)$ 

$$\ln\left(f_m^{(j)}(\eta)\right) = -(j+1)\eta - m\eta + ij\gamma\eta + \psi(\eta)$$

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Back to 
$$\ln\left(f_m^{(j)}(\eta)
ight)$$

$$\ln\left(f_m^{(j)}(\eta)\right) = -(j+1)\eta - m\eta + ij\gamma\eta + \psi(\eta)$$

$$= -j\eta \left[ 1 + O(j^{-1}) \right] - m\eta + ij\gamma\eta + j\eta \left[ 1 + O(j^{-1}) \right] - i\gamma j\eta \left[ 1 + O(j^{-1}) \right] + \eta \cdot m \left[ 1 + O(j^{-1}) \right] - i\gamma\eta \cdot m \left[ 1 + O(j^{-1}) \right] + j\eta^2 \left[ -\frac{(1 - \gamma^2)}{4} + i\frac{\gamma}{2} \right] \left[ 1 + O(j^{-1}) \right] + \eta^2 \cdot \frac{m^2}{j} \left[ \frac{1 - \gamma^2}{4} - i\frac{\gamma}{2} \right] \left[ 1 + O(j^{-1}) \right]$$

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$$f_{m}^{(j)}(\eta)$$

$$\frac{1}{J}\ln\left(f_{m}^{(j)}(\eta)\right)$$

$$\phi \rightarrow (\eta, J)$$

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Back to 
$$\ln\left(f_m^{(j)}(\eta)
ight)$$

$$\ln\left(f_m^{(j)}(\eta)\right) = -(j+1)\eta - m\eta + ij\gamma\eta + \psi(\eta)$$

$$= -j\eta \left[ 1 + O(j^{-1}) \right] - m\eta + ij\gamma\eta + j\eta \left[ 1 + O(j^{-1}) \right] - i\gamma j\eta \left[ 1 + O(j^{-1}) \right] + \eta \cdot m \left[ 1 + O(j^{-1}) \right] - i\gamma\eta \cdot m \left[ 1 + O(j^{-1}) \right] + j\eta^2 \left[ -\frac{(1 - \gamma^2)}{4} + i\frac{\gamma}{2} \right] \left[ 1 + O(j^{-1}) \right] + \eta^2 \cdot \frac{m^2}{j} \left[ \frac{1 - \gamma^2}{4} - i\frac{\gamma}{2} \right] \left[ 1 + O(j^{-1}) \right]$$

## The linear terms cancel

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Back to 
$$\ln\left(f_m^{(j)}(\eta)
ight)$$

$$\ln\left(f_m^{(j)}(\eta)\right) = -(j+1)\eta - m\eta + ij\gamma\eta + \psi(\eta)$$

$$= - m\eta + ij\gamma\eta \\ - i\gamma j\eta \left[1 + O(j^{-1})\right] \\ +\eta \cdot m \left[1 + O(j^{-1})\right] - i\gamma\eta \cdot m \left[1 + O(j^{-1})\right] \\ + j\eta^2 \left[ -\frac{(1 - \gamma^2)}{4} + i\frac{\gamma}{2} \right] \left[1 + O(j^{-1})\right] \\ + \eta^2 \cdot \frac{m^2}{j} \left[ \frac{1 - \gamma^2}{4} - i\frac{\gamma}{2} \right] \left[1 + O(j^{-1})\right] \\ + \eta O(1)$$

The linear terms cancel

▲ FNP



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Back to 
$$\ln\left(f_m^{(j)}(\eta)
ight)$$

=

$$\ln\left(f_m^{(j)}(\eta)\right) = -(j+1)\eta - m\eta + ij\gamma\eta + \psi(\eta)$$

 $-m\eta$ 

$$\begin{split} &+\eta \cdot m \left[ 1 + O\left(j^{-1}\right) \right] \ -i\gamma \eta \cdot m \left[ 1 + O\left(j^{-1}\right) \right] \\ &+ j\eta^2 \left[ - \frac{(1 - \gamma^2)}{4} + i\frac{\gamma}{2} \right] \left[ 1 + O\left(j^{-1}\right) \right] \\ &+ \eta^2 \cdot \frac{m^2}{j} \left[ \frac{1 - \gamma^2}{4} - i\frac{\gamma}{2} \right] \left[ 1 + O\left(j^{-1}\right) \right] \\ &+ \eta O\left(1\right) + i\gamma \eta O\left(1\right) \end{split}$$

The linear terms cancel

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Back to 
$$\ln\left(f_m^{(j)}(\eta)
ight)$$

=

$$\ln\left(f_m^{(j)}(\eta)\right) = -(j+1)\eta - m\eta + ij\gamma\eta + \psi(\eta)$$

$$\begin{split} &-i\gamma\eta\cdot m\left[1+O\left(j^{-1}\right)\right.\\ &+j\eta^{2}\left[-\frac{\left(1-\gamma^{2}\right)}{4}+\,i\frac{\gamma}{2}\right]\left[1+O\left(j^{-1}\right)\right]\\ &+\eta^{2}\cdot\frac{m^{2}}{j}\left[\frac{1-\gamma^{2}}{4}\,-\,i\frac{\gamma}{2}\right]\left[1+O\left(j^{-1}\right)\right]\\ &+\eta O\left(1\right)+i\gamma\eta O\left(1\right)+\frac{m}{j}\eta O\left(1\right) \end{split}$$

The linear terms cancel

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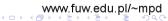
Finally  $\frac{1}{J} \ln \left( f_m^{(j)}(\eta) \right)$ 

Lets reorganise the quadratic terms:

$$\begin{split} \ln\left(f_m^{(j)}(\eta)\right) &= \left(1 + i\gamma + \frac{m}{j}\right) \eta O\left(1\right) \\ &- i\gamma\eta \cdot m\left[1 + O\left(j^{-1}\right)\right] \\ &+ j\eta^2\left[1 - \frac{m^2}{j^2}\right] \left[-\frac{(1 - \gamma^2)}{4} + i\frac{\gamma}{2}\right] \left[1 + O\left(j^{-1}\right)\right] \end{split}$$

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Finally  $\frac{1}{J} \ln \left( f_m^{(j)}(\eta) \right)$ 

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And finally divide everything by J

$$\frac{1}{J} \ln \left( f_m^{(j)}(\eta) \right) = \eta O\left(\frac{1}{J}\right) - i\gamma \eta \cdot \frac{m}{J} \left( 1 + O\left(\frac{1}{J}\right) \right) \\ + x\eta^2 \left[ 1 - \frac{m^2}{j^2} \right] \left[ -\frac{(1-\gamma^2)}{4} + i\frac{\gamma}{2} \right] \left( 1 + O\left(\frac{1}{J}\right) \right)$$

where  $x := \frac{j}{J} \in [0, 1]$ 

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Finally  $\frac{1}{J} \ln \left( f_m^{(j)}(\eta) \right)$ 

Lets reorganise the quadratic terms:

$$\begin{split} &\ln\left(f_m^{(j)}(\eta)\right) &= \left(1 + i\gamma + \frac{m}{j}\right) \eta O\left(1\right) \\ &\quad - i\gamma\eta \cdot m\left[1 + O\left(j^{-1}\right)\right] \\ &\quad + j\eta^2 \left[1 - \frac{m^2}{j^2}\right] \left[-\frac{(1 - \gamma^2)}{4} + i\frac{\gamma}{2}\right] \left[1 + O\left(j^{-1}\right)\right] \end{split}$$

And finally divide everything by J

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$$\frac{1}{J}\ln\left(f_m^{(j)}(\eta)\right) = -i\gamma\eta\cdot\frac{m}{J} + x\eta^2\left[1-\frac{m^2}{j^2}\right]\left[-\frac{(1-\gamma^2)}{4}+i\frac{\gamma}{2}\right] + O\left(\frac{1}{J}\right)$$

where  $x := \frac{j}{l} \in [0, 1]$ www.fuw.edu.pl/~mpd FNP EUROPEAN REGION ヘロト ヘアト ヘビト ヘン

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$$\phi_{\vec{m}}(\eta, J) = \sum_{i=1}^{N} \frac{1}{J} \ln \left( f_{m_i}^{(j_i)}(\eta) \right)$$

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$$\begin{split} \phi_{\vec{m}}(\eta, J) &= \sum_{i=1}^{N} \frac{1}{J} \ln \left( f_{m_{i}}^{(j_{i})}(\eta) \right) \\ &= O\left(\frac{1}{J}\right) - \eta \frac{i\gamma}{J} \sum_{i=1}^{N} m_{i} \\ &+ \eta^{2} \left[ -\frac{(1-\gamma^{2})}{4} + i\frac{\gamma}{2} \right] \sum_{i=1}^{N} x_{i} \left[ 1 - \frac{m_{i}^{2}}{j_{i}^{2}} \right] \end{split}$$

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$$\begin{split} \phi_{\vec{m}}(\eta, J) &= \sum_{i=1}^{N} \frac{1}{J} \ln \left( f_{m_i}^{(j_i)}(\eta) \right) \\ &= O\left(\frac{1}{J}\right) - \eta \frac{i\gamma}{J} \sum_{i=1}^{N} m_i \\ &+ \eta^2 \left[ -\frac{(1-\gamma^2)}{4} + i\frac{\gamma}{2} \right] \sum_{i=1}^{N} x_i \left[ 1 - \frac{m_i^2}{J_i^2} \right] \end{split}$$

Recall, that we consider only these matrix elements, that are SU(2) invariant, thus  $\sum_{i=1}^{N} m_i = 0$ , so the term linear in  $\eta$  vanish

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$$\begin{split} \phi_{\vec{m}}(\eta, J) &= \sum_{i=1}^{N} \frac{1}{J} \ln \left( f_{m_i}^{(j_i)}(\eta) \right) \\ &= O\left(\frac{1}{J}\right) - \eta \frac{i\gamma}{J} \sum_{i=1}^{N} m_i \\ &+ \eta^2 \left[ -\frac{(1-\gamma^2)}{4} + i\frac{\gamma}{2} \right] \sum_{i=1}^{N} x_i \left[ 1 - \frac{m_i^2}{J_i^2} \right] \end{split}$$

Recall, that we consider only these matrix elements, that are SU(2) invariant, thus  $\sum_{i=1}^{N} m_i = 0$ , so the term linear in  $\eta$  vanish

$$\phi_{\vec{m}}(\eta, J) = \eta^2 \left[ -\frac{(1-\gamma^2)}{4} + i\frac{\gamma}{2} \right] \sum_{i=1}^{N} x_i \left[ 1 - \frac{m_i^2}{j_i^2} \right]$$
(6)





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$$\begin{array}{l} f_{m}^{(j)}(\eta) \\ \frac{1}{J} \ln \left( f_{m}^{(j)}(\eta) \right) \\ \phi_{\overrightarrow{m}}(\eta, J) \end{array}$$

Hessian

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# Second derivative of $\boldsymbol{\phi}$

It is immediate to read  $\left(\frac{d^2\phi}{d\eta^2}\right)_{\eta=0}$  from (6)

$$\left(\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}\right)_{\eta=0} = \left[-\frac{(1-\gamma^2)}{2} + i\gamma\right]\sum_{i=1}^N x_i \left[1-\frac{m_i^2}{j_i^2}\right]$$

however we need to handle with the term  $\sum_{i=1}^{N} x_i \frac{m_i^2}{j_i^2}$ .

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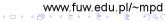
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 $\sum_{i=1}^{N} x_i \frac{m_i^2}{i^2}$ 

Note, that so far we had  $\phi_{\vec{m}}$  in basis  $|m_1,\ldots,m_N\rangle_{j_1\otimes\cdots\otimes j_N}$ . In this basis

$$\langle \vec{m} | \sum_{i=1}^{N} x_{i} \frac{m_{i}^{2}}{J_{i}^{2}} | \vec{m} \rangle_{\vec{j}} = \frac{1}{J} \langle \vec{m} | \sum_{i=1}^{N} \left( J_{(i)}^{2} \right)^{\frac{1}{2}} \frac{J_{z,(i)}^{2}}{J_{(i)}^{2}} | \vec{m} \rangle_{\vec{j}}$$

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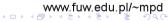
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 $\sum_{i=1}^{N} x_i \frac{m_i^2}{i_i^2}$ 

Note, that so far we had  $\phi_{\vec{m}}$  in basis  $|m_1, \ldots, m_N\rangle_{j_1\otimes \cdots \otimes j_N}$ . In this basis

$$\langle \vec{m} | \sum_{i=1}^{N} x_i \; \frac{m_i^2}{j_i^2} \; | \vec{m} \rangle_{\vec{j}} = \frac{1}{J} \; \langle \vec{m} | \sum_{i=1}^{N} \left( J_{(i)}^2 \right)^{\frac{1}{2}} \frac{J_{z,(i)}^2}{J_{(i)}^2} \; | \vec{m} \rangle_{\vec{j}}$$

But we are interested in matrix elements between invariant tensors  $|\iota\rangle$  , thus let's consider

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NOVATIVE ECONOMY

**FNP** 

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Hessian





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# Second derivative of $\boldsymbol{\phi}$

It is immediate to read 
$$\left(\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}\right)_{\eta=0}$$
 from (6)

$$\left(\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}\right)_{\eta=0} = \left[- \frac{(1-\gamma^2)}{2} + i\gamma\right] \sum_{i=1}^N x_i \left[1 - \frac{m_i^2}{j_i^2}\right]$$

however we need to handle with the term  $\sum_{i=1}^{N} x_i \frac{m_i^2}{j_i^2}$ , which appear to be equal

$$\sum_{i=1}^{N} x_i \ \frac{m_i^2}{j_i^2} = \frac{1}{3} \sum_{i=1}^{N} x_i$$

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so at the end of the day we have

$$\left(\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}\right)_{\eta=0} = \frac{2}{3} \left[- \frac{(1-\gamma^2)}{2} + i\gamma\right] \sum_{i=1}^N x_i$$

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# The final form of the Lorentzian polyhedra propagator

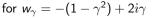
Given two basis elements  $\iota, \iota' \in \text{Inv} (\mathcal{H}_{j_1} \otimes \cdots \otimes \mathcal{H}_{j_N})$  the Lorentzian polyhedra propagator's matrix element is

$$\mathbb{T}_{\iota\iota'}=rac{1}{(4\pi)^2}\left(rac{6\pi}{w_{\gamma}\Sigma_{i=1}^N j_i}
ight)^{rac{3}{2}}~\delta_{\iota\iota}$$

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▲ ENP

 $\blacktriangleright$  The integrand  $\langle \iota | \ensuremath{\, Y^\dagger g} \ensuremath{\, Y} \, | \iota' \rangle$  has been studied

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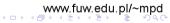
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- $\blacktriangleright$  The integrand  $\langle \iota | Y^{\dagger}g Y | \iota' \rangle$  has been studied
  - Smoothness of the exponent part has been proven thus the SPA is applicable



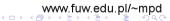
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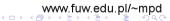
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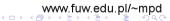
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  - On each space Inv ( $\bigotimes \mathcal{H}_{j_i}$ ) it is proportional to the identity with a factor dependent on total area of polyhedron  $A = \sum j_i$ .

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# Further directions

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Subleading order

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## Further directions

- Subleading order
- Applications to concrete examples

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## Further directions

- Subleading order
- Applications to concrete examples
- Boundary conditions

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## Thank you for your attention!

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